Collective effects of the cost of opinion change

Hendrik Schawe Laura Hernández

December 3, 2020

Opinion dynamics

evolution of opinions in a society of agents with time

- Social influence

  agents communicate and their opinion become more similar
- Homophily

  interaction happens between similar agents

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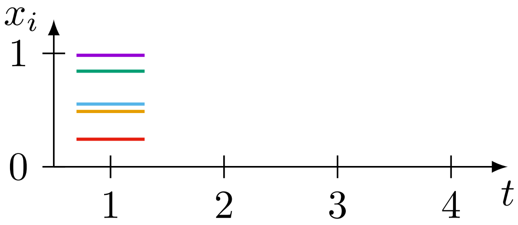
- Changing opinion (and therefore behavior) is not free

Hegselmann-Krause bounded confidence model

- N agents
- each with opinions $x_i \in [0, 1]$
- each with confidence $\varepsilon_i \in [0, 1]$
- interact with agent j, if $|x_i - x_j| \leq \varepsilon_i$
- compromise with your neighbors $x_i(t+1) = \frac{1}{|\mathcal{N}|} \sum_{j \in \mathcal{N}} x_j(t)$
- possible stationary states: *consensus* or *fragmentation*
- measure mean size of largest cluster $\langle S \rangle$ to detect consensus
- sharp threshold to consensus at $\varepsilon_i \approx 0.2$



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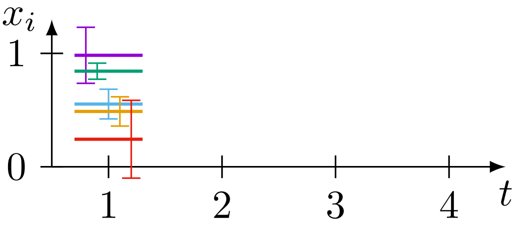

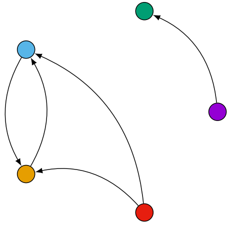

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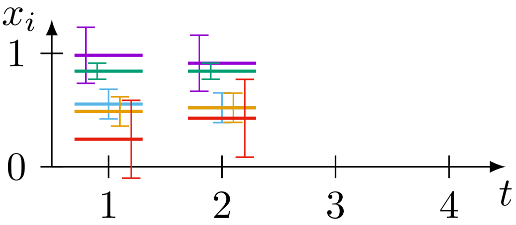

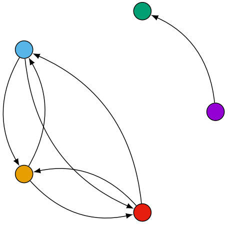

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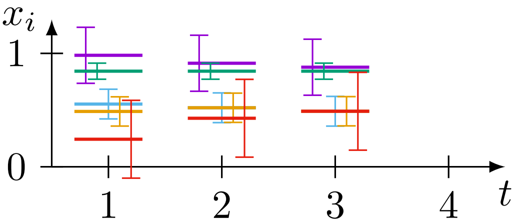

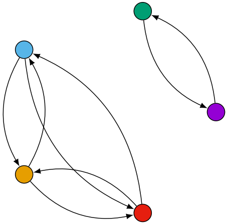

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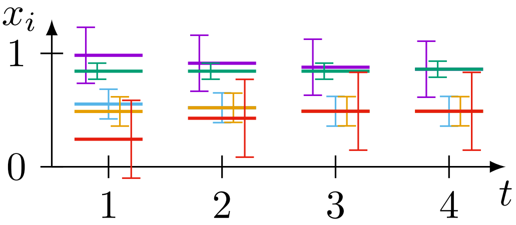

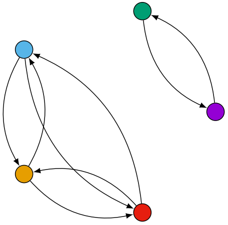

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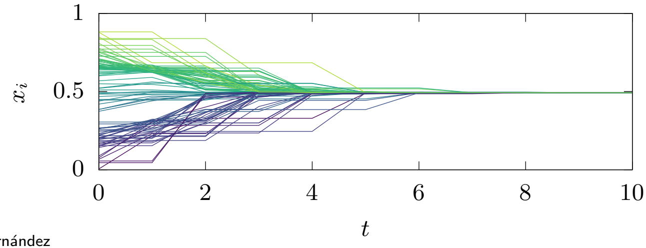

$N = 100, \varepsilon = 0.3$

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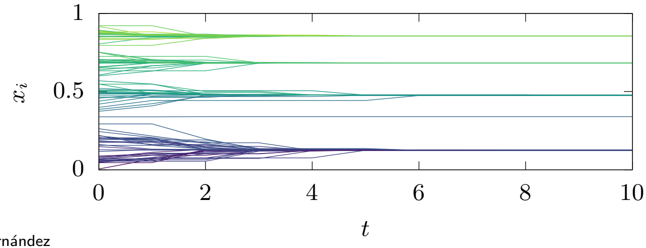
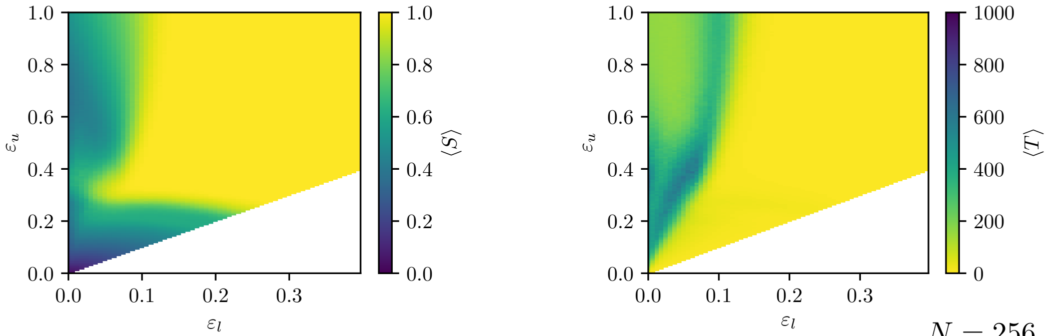


$N = 100, \varepsilon = 0.1$

Exploring the Landscape for $\varepsilon_i \in U[\varepsilon_l, \varepsilon_u]$



$N = 256$

- Phase space with nonmonotonous, complex structure
- Consensus where mean confidence $\varepsilon < 0.2$
- Surprising: Increasing confidence $\varepsilon_u \Rightarrow$ loss of consensus
- All effects are stronger with larger systems

When open mindedness hinders consensus, Schawe, Hernández, 2020, see also `youtu.be/_oSSz4olovE`

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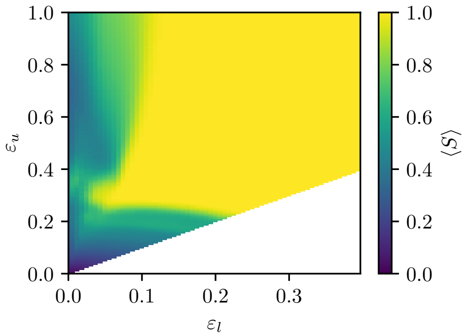

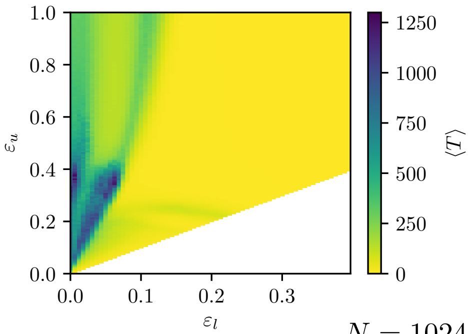

$N = 1024$

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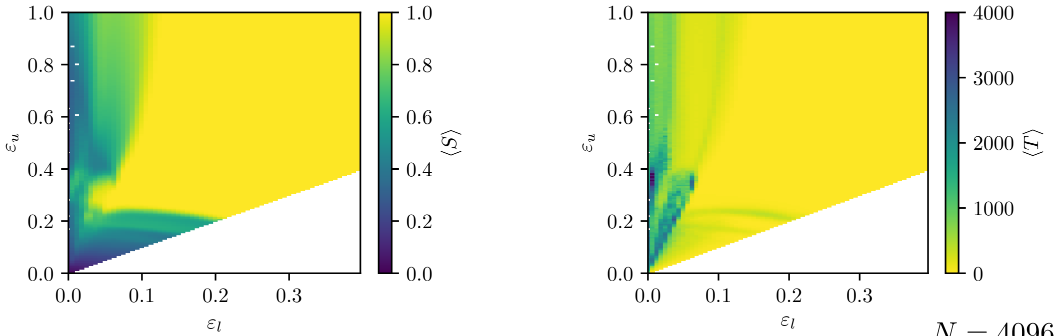

$N = 4096$

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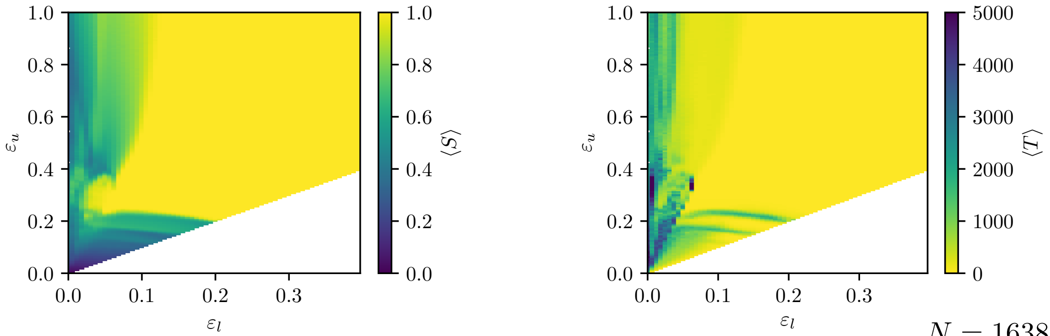

$N = 16384$

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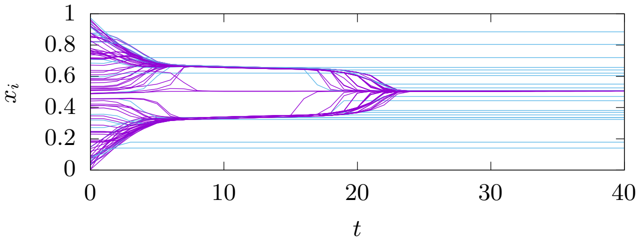
Adding Cost

- assign resources $c_i(0)$ to each agent
- changes cost proportional to opinion change
  $c_i(t+1) = c_i(t) - \eta\,|x_i(t) - x_i(t+1)|$
- opinions can not change without resources and *freeze*
- is there a critical cost?



$N = 16384$ (100 agents shown), $(\varepsilon_l, \varepsilon_u) = (0.1, 0.3)$, $\eta = 0.7$

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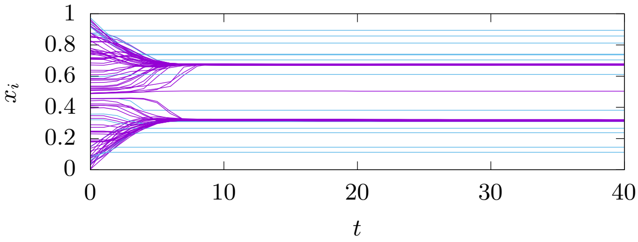
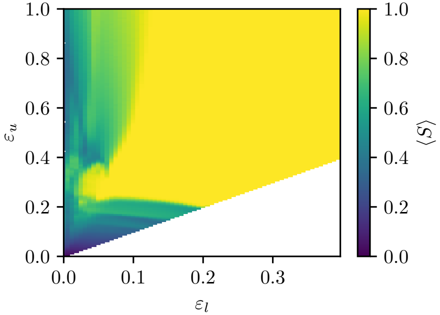
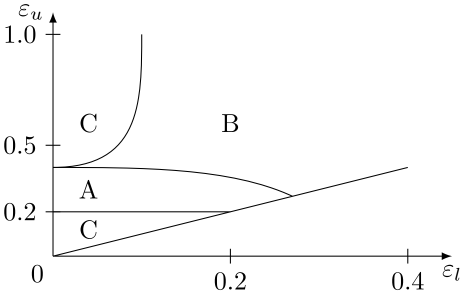


$N = 16384$ (100 agents shown), $(\varepsilon_l, \varepsilon_u) = (0.1, 0.3)$, $\eta = 1.1$

Different behavior in different parts of the $\varepsilon_l, \varepsilon_u$ space





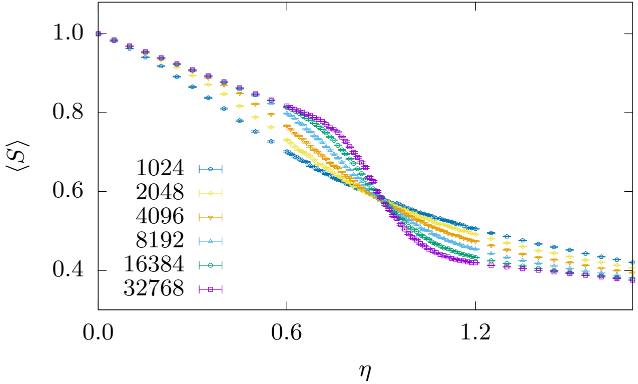
A. Phase transition from consensus to fragmentation at critical η

B. Always consensus, $\langle S \rangle$ shrinks linear in η

C. Never consensus

Region A: Second order phase transition

Finite-size scaling analysis



Collective effects of the cost of opinion change, Schawe, Hernández, 2020

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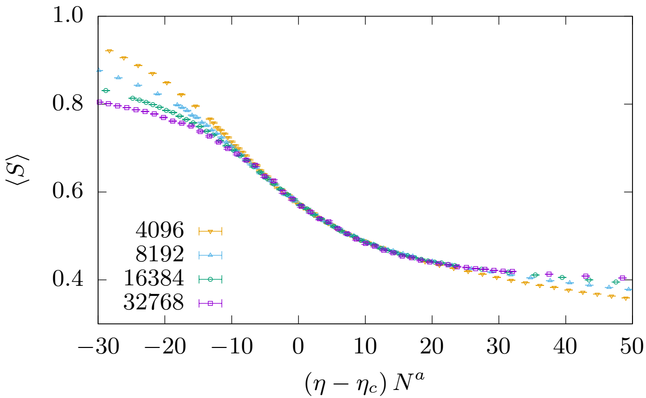

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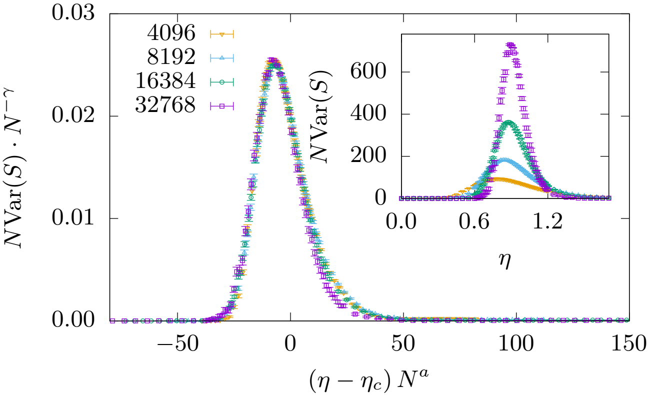

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Region A: Hints for universality

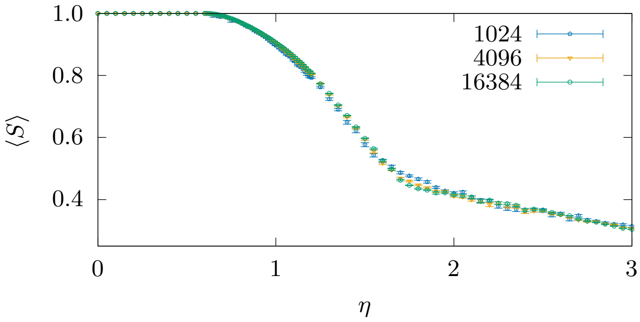
The exponent is robust for many different points within region A and ways to choose c_i

	$(\varepsilon_l, \varepsilon_u)$	η_c	a
$c_i(0) \in U[0,1]$	$(0.10, 0.30)$	$0.909(6)$	$0.45(4)$
$c_i(0) \in U[0,1]$	$(0.05, 0.30)$	$1.32(1)$	$0.42(5)$
$c_i(0) \in U[0,1]$	$(0.25, 0.25)$	$0.80(2)$	$0.41(6)$
$c_i(0) \in$ half-Gaussian	$(0.10, 0.30)$	$0.73(1)$	$0.43(2)$
$c_i \propto \varepsilon_i$	$(0.10, 0.30)$	$0.63(1)$	$0.43(1)$
$c_i(0) \in U[0.3, 0.7]$	$(0.10, 0.30)$	–	0

Region A: Hints for universality

The exponent is robust for many different points within region A and ways to choose c_i

But the phase transition vanishes if there are no very poor agents



$c_i(0) \in U[0.3, 0.7], (\varepsilon_l, \varepsilon_u) = (0.1, 0.3)$

Conclusions

- Simple model exhibiting complex behavior
- Introducing costs leads to a continuous phase transition from consensus to fragmentation
- Of relevance for society(?): If every agent can change its opinion to some degree, the sudden transition changes to a smooth crossover.

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Spoiler: Yes, of course.

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Spoiler: Yes, of course. But in an unexpected way!

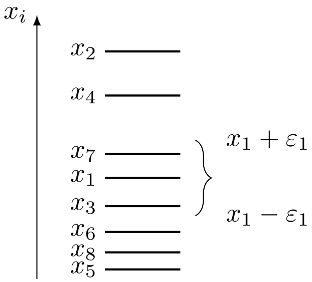
Appendix: Bonus Slides

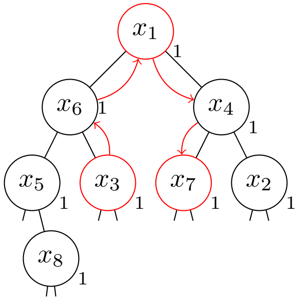
What is the problem when simulating? Introducing a faster algorithm.

- At each time step each agent has to average over all neighbors $\Rightarrow \mathcal{O}(N^2)$
- Introducing new algorithm
  - It is only necessary to touch the neighbors, which are far fewer for low ε_i
  - Converged clusters look for another agent like a single agent with high weight
- allows us to gather good statistics for systems two orders of magnitude larger $(N = 131072)$ than what is typically studied

Introducing a faster algorithm.

- Save all opinions in the system in a tree
- to average the neighbors of agent i
  - find the smallest opinion $x_j \geq x_i - \varepsilon_i$ in $\mathcal{O}(\log(N))$
  - traverse the tree in order and stop averaging on encountering $x_j \geq x_i + \varepsilon_i$
  - if a value x_j occurs more than once in the tree, assign it a weight

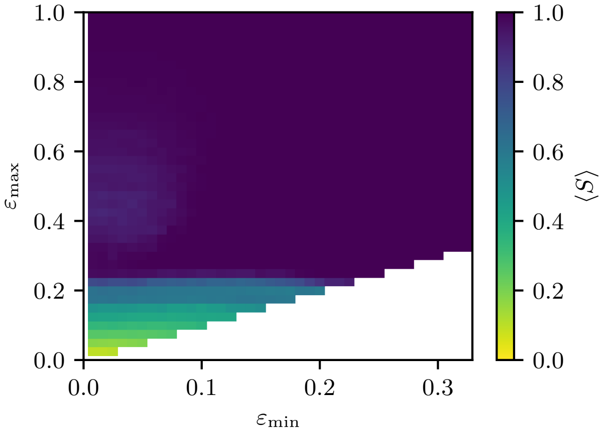




What about other distributions of ε_i?

Bounded power law

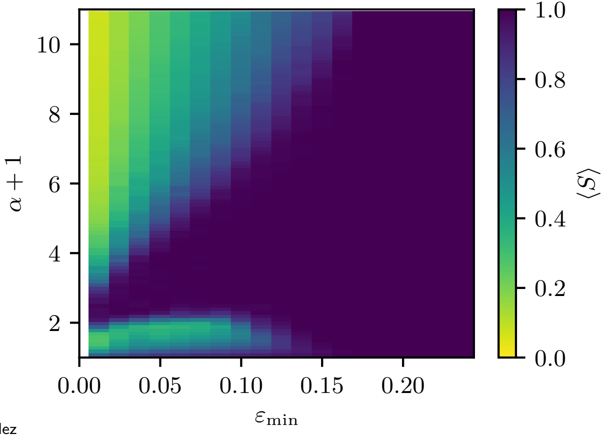
$$p(\varepsilon) = c\varepsilon^{-\gamma}$$



What about other distributions of ε_i?

Pareto

$$p(\varepsilon) = \frac{\alpha x_{\min}^{\alpha}}{x^{\alpha+1}}$$



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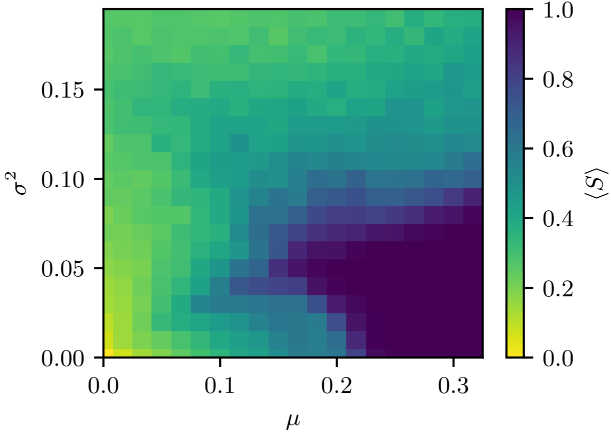
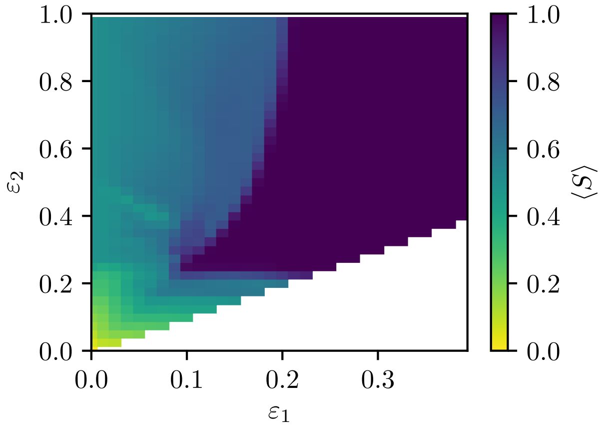
Gaussian



What about other distributions of ε_i?

Bimodal

$$p(\varepsilon) = \delta(\varepsilon - \varepsilon_1) + \delta(\varepsilon - \varepsilon_2)$$



Mean Dynamics