The Bridges to Consensus

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June 21, 2021

Watch the presentation at https://youtu.be/FYIRGbq-rIA

Introduction

- Opinion dynamics

  evolution of opinions in a society of agents with time
- Homophily (here: bounded confidence)

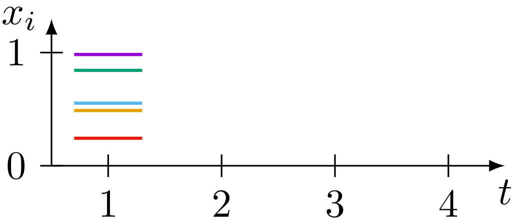
  agents influence only similar agents
- Social influence

  influence makes agents more similar

Can we observe complex emergent behavior?

Hegselmann-Krause bounded confidence model

- N agents
- each with opinions $x_i \in [0, 1]$
- each with confidence ε_i, but for our study $\varepsilon_i = \varepsilon$
- neighbors are topological neighbors on a static network which are also similar in opinion with $|x_i - x_j| \leq \varepsilon_i$
- compromise with your neighbors $x_i(t+1) = \frac{1}{|\mathcal{N}|} \sum_{j \in \mathcal{N}} x_j(t)$
- possible stationary states: *consensus* or *fragmentation*
- measure mean size of largest cluster $\langle S \rangle$ to detect consensus



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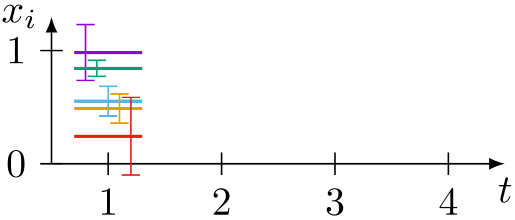

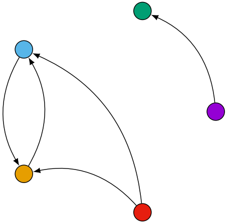

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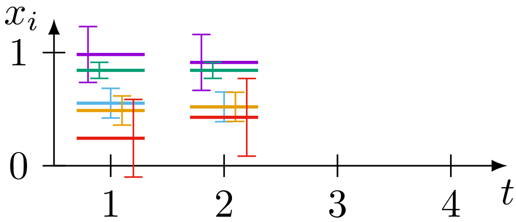

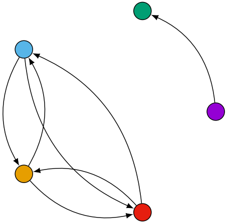

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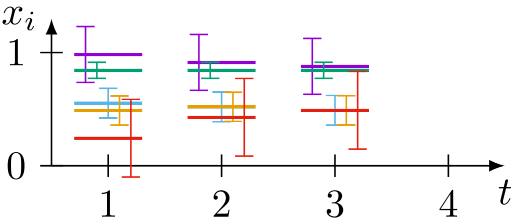

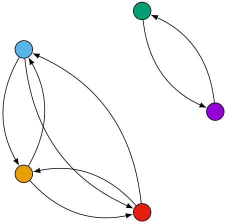

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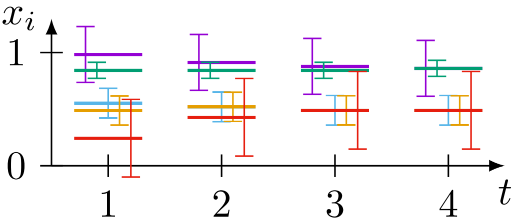

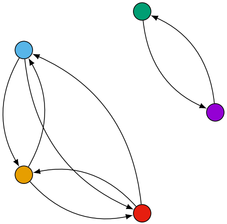

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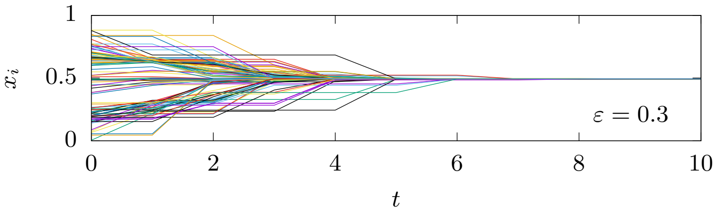

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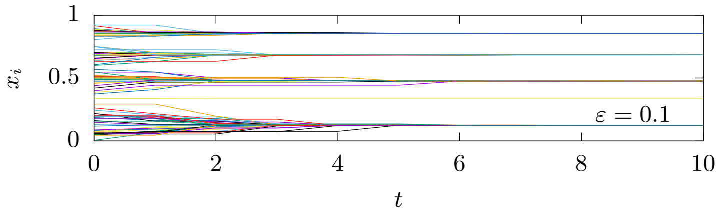


For which ε_i do we expect consensus?

Complete graph topology:

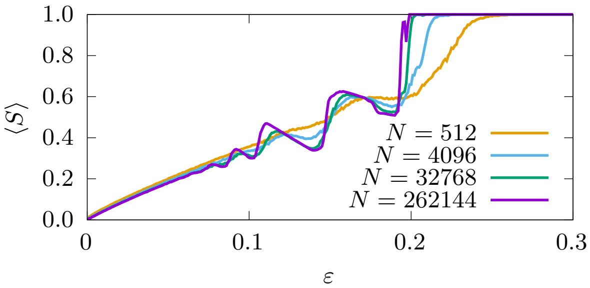
- $\varepsilon \gtrsim 0.2$ always consensus (for large N) [1]
- larger ε typically leads faster to consensus

Sparse topology:

- Unanimity threshold worsens for sparse topologies ($\varepsilon_c \sim 0.2 \rightarrow 0.5$) [2]
- Does the ability to reach consensus also deteriorate?
- Are there differences between lattices and random networks?

[1] Hegselmann, Krause, 2002, [2] Fortunato, 2004

The well known case: Mixed population



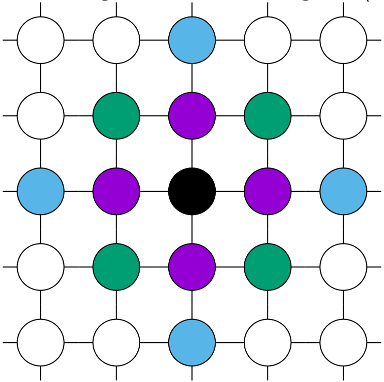
- Sharp transition at $\varepsilon_c = 0.1926(5)$
- *bifurcation* patterns: regions with m opinions

Largest systems simulated to date,
enabled by efficient algorithm [3]

[3] Schawe, Hernández, 2020

Lattices: A Lower critical value

Square lattice with third nearest neighbors, mean degree $\langle k \rangle = 12$



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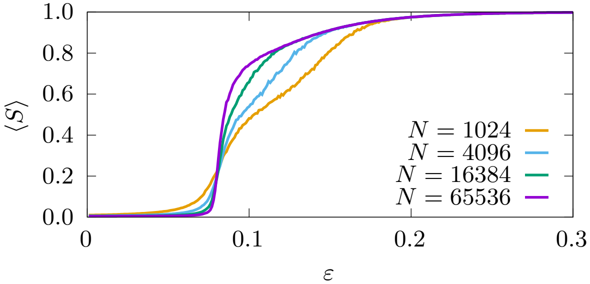
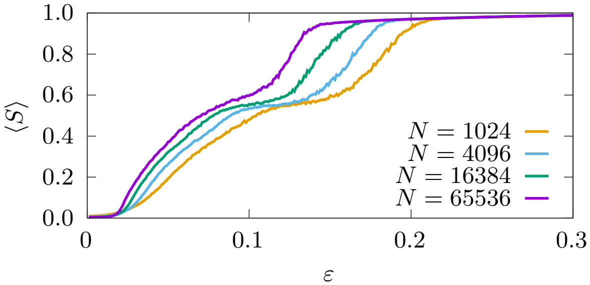


- still a sharp transiton but at much lower $\varepsilon_c = 0.0801(7)$
- unanimity threshold increases to $\varepsilon_u = 0.5$
- bifurcations vanish (i.e., no polarized state)

Random Networks: Bridges to consensus

Barabási Albert Graph with mean degree $\langle k \rangle = 10$



- crossover to consensus shifts as a power law to $\varepsilon_c = 0$
- unanimity threshold stays at $\varepsilon_u = 0.5$
- bifurcations vanish, polarization is preserved

$\Rightarrow$ For a sufficiently large system, there will be consensus

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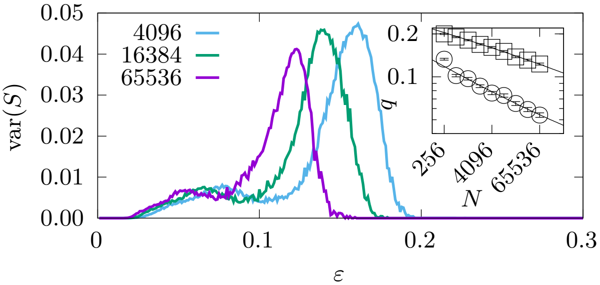

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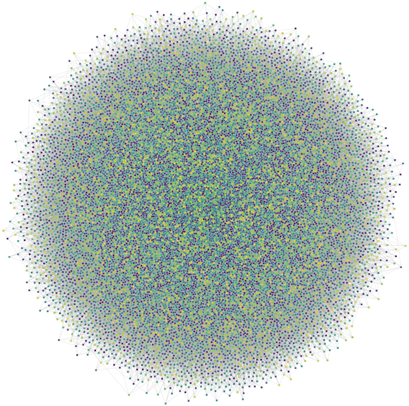
Random Networks: Bridges to consensus

How does this work?

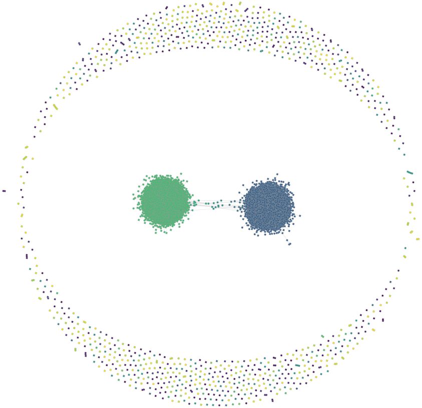
$t = 0$



Random Networks: Bridges to consensus

How does this work?

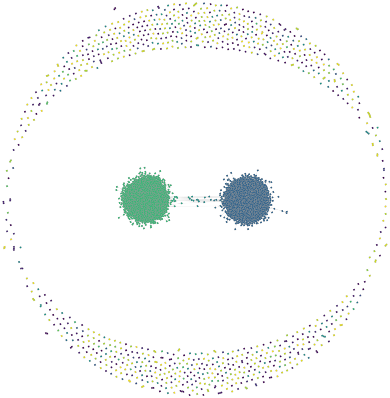
$t = 100$



Random Networks: Bridges to consensus

How does this work?

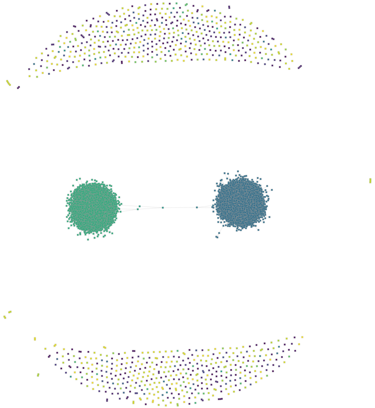
$t = 1000$



Random Networks: Bridges to consensus

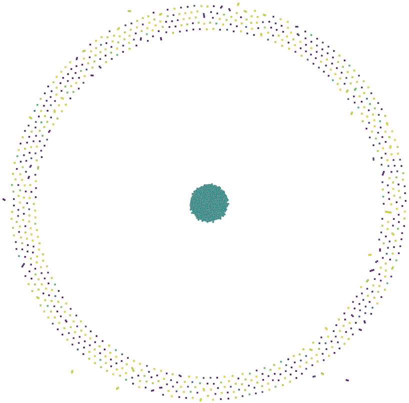
How does this work?

$t = 9000$



Random Networks: Bridges to consensus

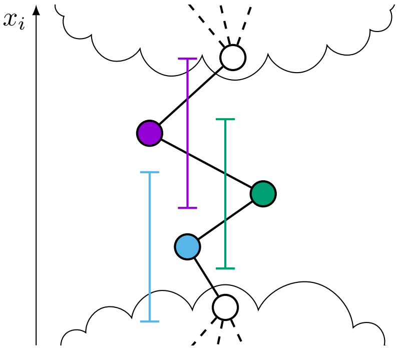
How does this work?
final configuration



Random Networks: Bridges to consensus

How does this work?

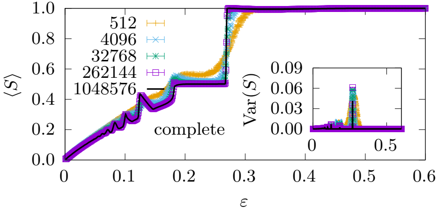
- synchronous updates enable long lived bridges
- over many iterations they pull the clusters together
- bridges are rare configurations, but one can be enough
- larger systems have higher probability to contain one

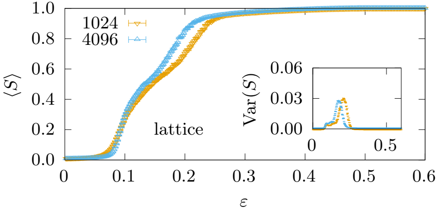


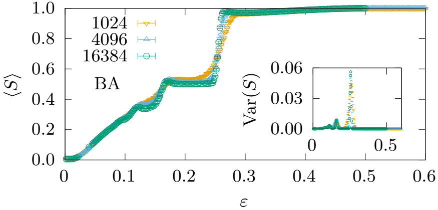
What about the Deffuant model?

It is the second famous bounded confidence model.

- sequential pairwise update excludes the possibility for bridges







Conclusions

- Sparse networks foster consensus at the cost of long convergence times
- mixed population, lattices and random networks show three otherwise fundamentally different behaviors
- find more details in Phys. Rev. Research **3**, 023208 (2021) (arxiv:2102.10910v2)
- raw data at `https://doi.org/10.5281/zenodo.4288672`

Appendix: Bonus Slides

What is the problem when simulating the mixed population?

- At each time step each agent has to average over all neighbors $\Rightarrow \mathcal{O}(N^2)$
- Introducing new algorithm [3]
  - It is only necessary to touch the neighbors, which are far fewer for low ε_i
  - Converged clusters look for another agent like a single agent with high weight
- allows us to gather good statistics for systems two orders of magnitude larger $(N = 262144)$ than what is typically studied

[3] Schawe, Hernández, 2020, code at `github.com/surt91/hk_tree`

Introducing a faster algorithm.

- Save all opinions in the system in a search tree (binary tree, B-tree, ...)
- to average the neighbors of agent i
  - find the smallest opinion $x_j \geq x_i - \varepsilon_i$ in $\mathcal{O}(\log(N))$
  - traverse the tree in order and stop averaging on encountering $x_j \geq x_i + \varepsilon_i$
  - if a value x_j occurs more than once in the tree, assign it a weight